M301, Exam 1

September 20, 2013

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Please show work when possible.

Name: ______________________________

THE EXAM HAS A TOTAL OF 100 POINTS
NO CALCULATOR OR PHONE USE ALLOWED
DO NOT OPEN THE QUESTION BOOKLET UNTIL THE EXAM BEGINS

1. In parts (a)–(b), consider the following system of linear equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_2 + 4x_3 &= 2 \\ 3x_2 + 6x_3 &= 3 \end{aligned}$$

(a) (10 points) Write the augmented matrix corresponding to the linear system, and compute its reduced row echelon form (RREF).

(b) (10 points) Is the system consistent? Either find a solution or explain why there isn't one.

2. (20 points) Consider the following chemical reaction:

$$x_1\,\mathrm{CH_4} + x_2\,\mathrm{O_2} \longrightarrow x_3\,\mathrm{CO_2} + x_4\,\mathrm{H_2O}.$$

Note that the elements involved in the reaction are Carbon (C), Hydrogen (H), and Oxygen (O). Use linear algebra to balance this equation; use smallest positive whole number coefficients if possible.

3. In each part (a)–(d), a mapping $T : \mathbb{R}^2 \to \mathbb{R}^2$ is described in terms of its action on arbitrary vectors with entries x_1 and x_2. In each case, either find the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^2$ or else give a reason why T is not a linear transformation.

   (a) (5 points) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_2 \\ -x_1 \end{bmatrix}$

   (b) (5 points) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 5 \\ x_2 \end{bmatrix}$

   (c) (5 points) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_2 \\ -x_1 - x_2 \end{bmatrix}$

   (d) (5 points) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

4. Suppose A is a 3×3 matrix such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{and} \qquad A \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}.$$

(a) (5 points) Are the columns of A linearly independent? Why or why not?

(b) (5 points) Is there a unique solution $\mathbf{x}$ to the equation $A\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$?

Why or why not?

(c) (5 points) Is there a unique solution $\mathbf{x}$ to the equation $A\mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$?

Why or why not?

5. Consider the vectors

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}.$$

(a) (10 points) Is the set $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ linearly independent?

(b) (10 points) Does the set $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ span $\mathbb{R}^3$?

(c) (5 points) Consider the matrix A whose columns are the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$. That is,

$$A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}.$$

Does the equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?